On the Use of Phased-Array Doppler Sonars Near Shore
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ABSTRACT

Phased-Array Doppler Sonars (PADS) have been used to probe an area several hundred meters on a side with 8 m spatial resolution, sampling every second or less with under 2 cm/s rms velocity error per sample. Estimates from two systems were combined to produce horizontal velocity vectors. Here concerns specific to use in shallow water are addressed. In particular, the shallower the water is, the larger the fraction of bottom backscatter, so the stronger the bias is toward zero Doppler shift in the estimates. First, direct comparisons are made with other current measurements made during the ONR-sponsored multi-investigator field experiment “SandyDuck,” which took place in fall 1997 off the coast of Duck, NC. While the coherences between PADS and in situ current measurements are high, the amplitude of the sonar response is generally low. To explore this further, a simplified model of wave shoaling is developed, permitting estimates of wave-frequency velocity variances from point measurements to be extrapolated over the whole field of view of the PADS for comparison. The resulting time-space movies of sonar response are consistent with quasi-steady acoustic backscatter intensity from the bottom competing with a variable backscatter level from the water volume. The latter may arise (for example) from intermittent injection of bubbles by breaking waves, producing patches of high or low acoustic response that advect with the mean flow. Once this competition is calibrated via the surface wave variance comparison, instantaneous measured total backscatter intensities can be compared to an estimated bottom backscatter level (which is updated on a longer time-scale, appropriate to evolution of the water depth or bottom roughness) to provide corrected sonar estimates over the region.

1. Introduction

The flows near shore are forced by waves and wind, with additional influences due to the larger scale context (tides, outflow from inlets, shelf waves, etc.). The dynamics near shore also involve bathymetry, with wave refraction (Kaihatu and Kirby, 1995; Kennedy et al., 2000; Lippmann et al., 1996; Raubenheimer et al., 1996), “channeling” of the mean flows (Shepard and Inman, 1950), and the movement of sediment (Inman et al., 1971; Inman and Brush, 1973; Shepard and Inman, 1950). An understanding of the form and dynamics of these interactions near shore could lead to predictions of instabilities and rip currents (Allen et al., 1996; Bowen and Holman, 1989; Oltman-Shay et al., 1989; Ozkan-Haller and Kirby, 1999; Reniers et al., 1997; Slinn et al., 1998), of the net effects on horizontal mixing and diffusion (Inman et al., 1971), and of the feedback on morphological evolution and beach erosion (Holman, 1995; Holman and Bowen, 1982). The fluxes of momentum, vorticity, and mass (including sand, bubbles, nutrients, eggs, larvae, pollutants, etc.) across the nearshore region are important in the larger-scale perspective too. Useful parameterization of these fluxes could be viewed as important “output” from an understanding of the nearshore dynamics, as well as a good measure of how well the dynamics are understood.

A significant requirement for studying the nearshore system is an ability to observe both the incident waves and the underlying flow. Ideally, one would like to resolve gradients in wave-related quantities (mass-flux, radiation stress; e.g., see Longuet-Higgins and Stewart 1962, Longuet-Higgins and Steward 1964, Longuet-Higgins 1970) over an area while simultaneously resolving the associated currents over a wide range of scales in time and space. Vorticity associated with the flow would be useful as both a diagnostic of the dynamics and a critical link to modeling. However, from a practical point of view, vorticity and flux gradients involve spatial derivatives, which are hard to extract from observations.

Acoustic techniques show promise for such extensive probing of the region near shore. For example, a study of “rip currents” was carried out with an earlier version of PADS (Smith and Largier, 1995).
“Blocking” by the bubble plumes left by plunging breaker was described for systems from 40 kHz to 200 kHz (Smith, 1993; Thorpe and Hall, 1993), and appears to limit applicability to the region outside the break-point of incoming surf. More recently, influences of surf-generated bubbles on acoustic propagation (Farmer et al., 2001) and of bubble advection in the surf-zone (Dahl, 2001) and in rip currents (Caruthers et al., 1999; Vagle and Farmer, 2001) have been described. However, biasing of Doppler-based velocity estimates toward zero by bottom interference has not been quantitatively addressed previously.

A companion paper (Smith, 2001) describes the general use of high-frequency “phased-array Doppler sonars” (PADS) to probe near-surface horizontal velocities over a continuous time-space segment extending hundreds of meters on a side and many hours in duration. Given the comprehensive measurement needs mentioned above, application of this technique in the nearshore is compelling. Continuous coverage of waves and currents in space and time appears feasible, permitting estimation of the vertical component of vorticity of the nearshore currents, and of the divergence of the waves’ “radiation stress” and mass flux (for example). Smith (2001) describes the essential technique and algorithms yielding quantitative estimates of errors and biases, including an objective technique for combining information from two (or more) systems to estimate horizontal vector velocities.

Here concerns specific to using high-frequency (~200 kHz) PADS in shallow water are addressed. There are several aspects of the nearshore environment that distinguish it acoustically from deep water: (1) The bottom backscatters sound that competes with the signal from scatterers in the water volume. The received signal attributable to bottom backscatter varies with water depth, and can also vary slowly in time (presumably as bottom roughness characteristics evolve). In general, this becomes significant when the wind and waves are weak, when few bubbles are generated. (2) Plunging breakers can produce a “wall” of bubbles so dense it is acoustically impenetrable (Smith, 1993; Thorpe and Hall, 1993). This limits the shoreward extent of measurements, confining the sample area to outside the active surfzone. (3) There are large variations in the scatterer content of the water, on scales of meters to tens of meters (e.g., water advecting offshore in “rip currents” that is full of bubbles from the surfzone (Smith and Largier, 1995)). (4) Advection of water from inlets can also lead to variations in stratification and in particle content of the water.

These issues are addressed through comparisons with data collected as part of “SandyDuck,” a multi-investigator field experiment sponsored by ONR with assistance from the Army Corps of Engineers and the USGS. Data in or near the field of view of a dual-PADS deployment were provided by P. Howd (vertical profiles at one location, 3 min. average currents) and by S. Elgar, R. Guza, T. Herbers, and W. O’Reilly (current meters at many locations, near bottom; 0.5 s samples). The selection of comparisons and interpretation of the results are guided by consideration of the underlying physics. In particular, the predictable behavior of surface waves propagating on a nearly planar beach permits the comparisons to be extended over the whole area, and to the full time-space behavior of the response. The results indicate a relatively straightforward competition between bottom and volume backscatter.

2. Experimental Setup

Two “Phased Array Doppler Sonars” (PADS) were deployed as part of “SandyDuck” in September and October, 1997 at the Field Research Facility (FRF) of the US Army Corps of Engineers. Looking shoreward from the 6-m depth contour, they probed a total area about 400 m alongshore by 350 m cross-shore (figure 1). Over the smaller region probed by both systems, perhaps 200 m by 300 m, horizontal velocity vectors are fully resolved. In the outer corners, only one component is resolved; however these 1-component estimates still provide useful information, particularly concerning wave propagation.
The Doppler processing, error estimation, and method for combining information from two (or more) PADS is discussed in a companion paper (Smith, 2001). Briefly, the approach is similar to the use of “dual-Doppler radars” to map winds, but using sound and covering a smaller area with higher resolution. An acoustic signal is projected in a wide horizontal fan, radiating outward in the water from the instrument package and filling the water column in shallow water. The sound scatters off particles in the water (especially bubbles) and off the bottom. Some backscattered sound returns to the sonar, where the signal is received on an array, beamformed into returns from various directions, and analyzed for frequency shift versus direction and elapsed time since transmission. For direct-path transmission and return, the time-delay since transmission translates to distance from the sonar. The frequency shift of the backscattered signal (Doppler shift) is proportional to the radial component of the velocity of scatterers at the sample volume. The systems discussed here were operated at 190 kHz and 225 kHz center frequencies, with 11 repeats of two different 13 bit Barker codes to spread the signal over 15.6 kHz bandwidth (64 µs “bits,” sampled every 32 µs; see Pinkel and Smith 1992; Smith 2001). The measurements are resolved to 7.6 m (range) by 6 degrees (bearing), with new estimates produced every 0.75 seconds, pair-averaged to 1.5 second sample rate. The resulting velocity “radials” have rms error levels of order 1.5 cm/s (Smith, 2001). By combining the radial velocities from two
such devices located some 300 m apart (figure 1), both horizontal components of velocity can be estimated on a grid several hundred meters on a side.

Vertical location (elevation angle) is not resolved; the ~22° vertical beam-width takes in the whole water column, and the effective location of the measurements is dictated by the centroid of scatterers. In the frequency range considered here (175 kHz to 240 kHz), microbubbles are efficient scatterers. These are produced copiously by breaking waves, and in general dominate the backscatter even outside the surfzone when the wind exceeds 5 m/s or so (inside the surfzone, and in rip currents carrying surfzone water offshore, microbubbles are ubiquitous). In deep water, bubble densities vary by orders of magnitude over moderate horizontal distances (depending strongly on windspeed), and have a mean vertical distributions approximated by an exponential with a depth scale of order 1 to 2 m, depending weakly on windspeed (Crawford and Farmer, 1987; Thorpe, 1986). In shallow water this distribution may be different, and one issue here is the effective depth of the measurement.

3. Comparisons with Current Meter Data

a. Low frequency comparisons.

Recent studies with similar Doppler sonars used at grazing angles indicate that the velocity estimates correspond to Eulerian velocities measured some depth below the wave troughs (Smith, 1998). It is therefore appropriate to compare the low-frequency sonar estimates directly with those from current meters or profilers. This would also indicate that the low-frequency comparison is not sensitive to non-linearity of the waves. The a priori hypothesis is that the effective depth of the sonar measurements is of order 1.5 m below the mean surface for typical oceanic conditions (~1 m waves, bubble layer with 1.5 m scale depth).

Comparisons are made first using velocities averaged over 1 to 5 minutes. For the resolved frequencies (low compared to the incident surface waves), stratification can be important. Thus two

![Figure 2. Correlations between PADS data at the profile location and the horizontal currents measured at various depths. The stratified case (10/13/1997) exhibits a broad maximum near 1.5 m depth. The unstratified case (10/15/1997) yields higher correlations with little if any depth dependence.](image-url)
time periods with contrasting conditions are examined: (1) 1600 to 1800 UTC, 10/13 was a calm period with moderate stratification of the water column; and (2) 2100-2400 UTC 10/15 was a period with moderate winds and a well-mixed water column.

At one location, 3-minute averaged current profiles were available (courtesy of P. Howd, Univ. South Florida). Figure 2 shows the correlations (means included) between PADS measurements versus the horizontal velocity at each depth, computed over the 2 hours (on 10/13) or 3 hours (10/15) of available data. In the stratified case (data from 10/13) the correlation varies with depth, providing one indication of the effective depth of the PADS measurements (not dissimilar from the a priori estimate). The well-mixed case (data from 10/15) shows better agreement between the PADS and currents at every depth measured (to 0.75 m from the bottom).

Comparisons were also made with data available from several other locations, but limited to current meters about 0.5 m off the bottom (courtesy of S. Elgar (WHOI), R.T. Guza (SIO), T. Herbers (NPS), and W. O’Reilly (NPS)). For stratified cases, there can be substantial changes in current direction and magnitude between the surface and bottom (see figure 1), and this can compromise comparisons made at these frequencies (low relative to surface wave frequencies).

b. Wave Frequency Correlations

To obviate effects of stratified flow, comparisons were made over the frequencies of the incoming waves. Suitable data, sampled at 2 Hz, were provided by S. Elgar (WHOI), R.T. Guza (SIO), T. Herbers (NPS), and W. O’Reilly (NPS) from “Sonic altimeter, Pressure, and U, V current measurement frames (“SPUVxx”, where “xx” is an ID number). The SPUV current measurements are roughly 0.5 m above the bottom. In the comparison area (outside the surfzone), the depth dependence of motion at wave frequencies is both moderate and well understood. Unfortunately, the support frames for the current meters produced significant acoustic interference. Fortunately, quantitative comparisons are usefully made between the current meter data and PADS estimates nearby (~15 m away), including both correlations and “scale factors” relating the two kinds of data (see Table 1). These comparisons were carried out at 15 sites within the field of view of both PADS (see figure 1). The essence of the comparison is demonstrated from a deeper site and two shallower (Table 1). It was found that the correlations between PADS velocities and the current meters are high, similar to those for similarly separated current meter pairs (Table 1, last column). However, while the correlations are

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<td>0.855</td>
<td>1.75</td>
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<td>0.943</td>
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Table 1. Wave-frequency correlations and scale-factors between PADS and current meters (SPUVs, see text). Locations are given in FRF coordinates (x increases offshore, shoreline is near 110 m; y increases alongshore to the NNW; see figure 1). The magnitude of correlation between two SPUVs separated by 13 m (last column) is comparable to the correlation between the PADS and SPUV. Due to acoustic interference, PADS estimates from about 15 m away are used in the comparisons (see text). The scale factor required to bring PADS velocities up to the SPUV levels is always greater than 1. The systematic difference in scale between SPUV62 and SPUV63 may be due to undulations of the bottom or partial sheltering by the FRF pier (located 300 m to the SSE).
The scaling factors up to 3.73 are needed to match the current meter magnitudes. Further, the scaling adjustment at the deeper site (Table 1, first column) is always smaller than at shallower sites. This suggests that the cause may be interference from bottom backscatter (having zero or near-zero Doppler shift), which has increasing effect as the water depth decreases.

The correlations and scale factors were investigated as functions of frequency as well (cross-spectra and transfer functions). Within the limits of the resolved wave field, these appear to be uniform across frequency. This suggests that such wave-frequency comparisons can be used to “calibrate” the PADS estimates independent of frequency. However, we desire such a calibration over the whole field of view, not just at a few isolated points.

4. A Simplified Wave Model

In this section a simple model is described for the evolution of waves as they shoal. Given values of wave properties at one or a few specified locations (e.g., from the SPUV arrays or at the 8 m array operated by C. Long of the FRF; see Long and Oltman-Shay 1991), this model provides estimates of the radial velocity variances over the whole field of view of each PADS. For this purpose, the simplest description that captures the essence of the wave behavior is sought. Wave-frequency variances formed from averages over a few minutes from the PADS measurements can then be compared to the model-extrapolated estimates, providing statistical views of the sonar response as it evolves in time and space. The variances of velocity and elevation, and the relationships between them, are not sensitive to wave non-linearity (in contrast to, e.g., skewness or asymmetry). Thus, a second-order description of the waves is adequate, which is a sufficient condition for action conservation to hold (Whitham, 1974).

For a beach that is approximately uniform in the alongshore direction, the wave description reduces to a 1D problem, based on refraction, action conservation, and dissipation (cf., Thornton and Guza 1986). Currents other than the orbital velocities of the waves are neglected. An adequate set of input parameters is provided by the depth profile \( h(x) \) and wave properties at a specified depth (say at 8 m depth); e.g., period \( T \), a measure of wave amplitude such as significant wave height \( H_s \), principle propagation direction \( \phi \), and a measure of the directional spread. The waves are assumed to be narrowband, so that a description based on the dominant period and direction is sufficient. In general, the mean wave propagation direction \( \phi \) will tend toward the shoreline, but be incident at some angle off normal (not quite parallel to the \( x \) axis, defined as shore-normal).

For waves that are statistically steady and homogeneous in \( y \), action conservation reduces to

\[
\frac{\partial}{\partial x} (c_x^s A) = 0
\]  

(1)

where \( c_x^s \) is the \( x \)-component of group velocity and \( A \) is the action density. Dissipation is neglected to simplify exposition; however it is straightforward to implement the semi-empirical dissipation rate described by Thornton and Guza (1986) (for example), and this was done for the data comparisons.

Wave action \( A \) is the ratio of wave energy to frequency, \( A = E/\sigma \), where \( E = \rho g H_s^2 / 32 \). For statistically steady waves and no currents, the wave frequency \( \sigma = 2\pi/T \) is constant. Gravity \( g \) and water density \( \rho \) are also constants, so \( A \) is simply proportional to \( H_s^2 \). Thus 1 integrates to

\[
c_x^s H_s^2 = c_x^s H_1^2 \cos \phi = \text{constant}
\]  

(2)
\[ H_s^2(x) = H_0^2 \left( \frac{c_s^0 \cos \phi_0}{c^g(x) \cos \phi(x)} \right) \]  

(3)

where \( c_s^0, H_0, \) and \( \phi_0 \) are the values at the reference location.

To obtain \( c^g \) and \( \phi \) as functions of \( x \), first the wavenumber magnitude \( k(x) \) is obtained (iteratively) from the linear dispersion relation for gravity waves in finite water depth,

\[ \sigma^2 = gk \tanh kh, \]  

(4)

given the frequency \( \sigma \) and water depth \( h(x) \). The group speed is

\[ c^g \equiv \frac{\partial \sigma}{\partial k} = \frac{1}{2} c_p \left( 1 + \frac{2kh}{\sinh 2kh} \right) \]  

(5)

where \( c_p \equiv \sigma/k \) is the phase speed. Since the system is uniform in \( y \), conservation of wave crests implies that the \( y \)-component of wavenumber \( k_y \) is constant (Phillips, 1977). Then the propagation angle \( \phi \) varies according to

\[ \phi = \arcsin \left( \frac{k_0}{k} \sin \phi_0 \right) \]  

(6)

where \( k_0 \) is the wavenumber at the reference location. Note that, particularly if the reference location is in shallower water than the target area, this can produce complex values. This implies an imaginary value of \( k_x \), or an edge-wave-like solution. In the present context this does not cause problems.

The objective is to estimate velocity variances. It is instructive to consider a set of correlations between the Cartesian components of velocity in “beach coordinates,” \( \langle V_x^2 \rangle, \langle V_y^2 \rangle, \) and \( \langle V_x V_y \rangle \), rather than wave height, direction, and directional spread (say). Here \( \langle \rangle \) denotes an appropriate average (e.g., over a few minutes). The principle wave direction can be found from the major axis of variability:

\[ \phi = \frac{1}{2} \arctan \left( \frac{2 \langle V_x V_y \rangle}{\langle V_x^2 \rangle - \langle V_y^2 \rangle} \right). \]  

(7)

The 180° ambiguity in direction hardly matters since the velocity variances are symmetric about wave propagation direction, but it is safe in this context to take the principle direction to be angling toward the shoreline. Total horizontal velocity variance is

\[ \langle V_T^2 \rangle \equiv \langle V_x^2 \rangle + \langle V_y^2 \rangle, \]  

(8)

which can be related to the wave height and period via linear theory. In finite depth water, we obtain:

\[ \langle V_T^2 \rangle = \frac{g^2 k^2 a^2}{\sigma^2} = \left( \frac{gk}{\sigma} \right)^2 \left( \frac{1}{4} H_s \right)^2 = \left( \frac{gkH_s T}{8\pi} \right)^2 \]  

(9)

The component velocities in coordinates aligned with the wave propagation direction \( \phi \) are
\( V_\parallel = V_x \cos \phi + V_y \sin \phi \) and \( V_\perp = V_y \cos \phi - V_x \sin \phi \), \hspace{1cm} (10)

leading to the wave-aligned variances

\[
< V_\parallel^2 > = < V_x^2 > \cos^2 \phi + < V_y^2 > \sin^2 \phi + < V_x V_y > \sin 2\phi
\]

\hspace{1cm} (11)

and

\[
< V_\perp^2 > = < V_x^2 > \sin^2 \phi + < V_y^2 > \cos^2 \phi - < V_x V_y > \sin 2\phi
\]

\hspace{1cm} (12)

Note that \( V_\parallel^2 + V_\perp^2 = V_T^2 \). Also, by construction, the cross-correlation between components is zero in these coordinates: \( < V_\parallel V_\perp > = 0 \). Indeed, if the waves are unidirectional then \( V_\perp \equiv 0 \). In contrast, if the waves propagate in all directions equally (isotropic), then \( V_\parallel^2 = V_\perp^2 = \frac{1}{2} V_T^2 \). So a relevant measure of the directional spread is given by

\[
D \equiv < V_\parallel^2 > / < V_T^2 > ,
\]

\hspace{1cm} (13)

which becomes 1.0 for unidirectional waves, and falls to 0.5 for isotropic waves. In general the results are not sensitive to the value, and it can be set to a value near 0.9 for most nearshore conditions.

These wave-aligned variances provide a simple form for evaluating “radial component” variances \( < V_R^2 > \), measured along an arbitrary direction \( \theta \):

\[
< V_R^2 > = < V_\parallel^2 > \cos^2 (\phi - \theta) + < V_\perp^2 > \sin^2 (\phi - \theta)
\]

\[
= < V_T^2 > \left( D \cos^2 (\phi - \theta) + (1 - D) \sin^2 (\phi - \theta) \right)
\]

\hspace{1cm} (14)

**Figure 3.** Diagram illustrating the evolution of component velocity variance vs. measurement angle between two depths. A directional distribution of 10.5 s period waves is transformed from an initial position in 8 m water depth to 5 m depth using the simplified wave model (see text). Initial values (8 m depth): angle 9°, \( H_s = 1.0, D = 0.70 \) (inner, darker “pinched oval” with darker principal axis). At 5 m depth: angle 7.26°, \( H_s = 1.6, D = 0.78 \) (outer, lighter pinched oval with lighter principal axis).
This form appears apt for the PADS data, since the velocity estimates are produced in arrays corresponding to a set of fixed angles by a set of fixed ranges. However, note that the directional spread (and hence $D$) varies as waves shoal, so this is applicable only with the additional approximation that $D$ does not vary significantly.

A simple but improved approximation can be obtained in beach coordinates, taking advantage of the invariance of $k_y$. The covariances in beach coordinates can be written

$$< V_i^2 > = \left( \frac{g}{\sigma} \right)^2 < a^2 > < k_x^2 > = (const) < H_i^2 > = (const) < H_x^2 > k_x^2 \cos^2 \varphi$$  (15)

**Figure 4.** Range-angle maps of the ratio of estimated RMS wave orbital velocities parallel to the sonar beam ($V_e$) to measured RMS values ($V_m$), for a day with low backscatter. The bottom contours illustrate the distortion due to viewing in range-angle coordinates (compare figure 1). Green denotes equal variances, white denotes a measured variance of half the estimated value, tan means they are 1/4, and red means 1/8 or smaller. The lines extending upward from range index 20 in each panel denote the angle parallel to principle wave propagation direction. Over most of the areas covered by each sonar, the measured variances are systematically smaller than the estimated “true” values. The bias toward zero generally increases as the water depth decreases, until the signal fades at maximum range (near index 80 to 90). Backscatter from frames at the instrumented sites also reduces the measurement response. Range increments are 3.8 m, so the maximum range is about 418 m. Angle increments are 1.98° (far) and 1.76° (near), so the net angular spans are 128° (far sonar) and 113° (near sonar).
\[
<V_y^2> = \left(\frac{g}{\sigma}\right)^2 <a^2> <k_y^2> = (\text{const}) <H_y^2>
\] (16)

and
\[
<V_x V_y> = \left(\frac{g}{\sigma}\right)^2 <a^2> <k_x k_y> = (\text{const}) <H_x^2> <k_x> = (\text{const}) <H_x^2> k \cos \varphi .
\] (17)

The approximations in 15 and 17 depend on the variations of \(k_x\) being smaller than its mean, \(<k_x>\). Indeed, the evolution equations for \(a^2\) (3) and \(\varphi\) (6) implicitly contain similar assumptions. Near shore, this is a good assumption, since the wave direction is generally toward the shore. The results are expressed in terms of quantities for which simple solutions are described above, and can be translated back into wave-coordinates via 7, 11, 12, and (for \(D\)) 13 (see figure 3). Using these, 14 provides estimates of radial velocity variances that capture the essential behavior of the waves, as desired. Evolution of the complete frequency-directional spectrum need not be considered in detail.

Figures 4 and 5 illustrate the estimates of rms velocity magnitude due to the incident waves \(V_e\) divided by that measured \(V_m\), for the component of velocity along each sonar direction resolved. The ratios are presented as color-contoured maps of \(\log_2(V_e/V_m)\) versus range and angle. In calm conditions

![Figure 5](image.png)

**Figure 5.** As in figure 4, but for windier conditions, when there are lots of scatterers (bubbles) in the water volume. The zero bias (thought to be due to bottom interference) is much smaller in this case.
(figure 4), the measured velocities are significantly smaller than those estimated for the incident waves. The discrepancy increases in shallower water. In windier conditions (figure 5) the discrepancy is much smaller, with ratios generally smaller than 2 (except very near shore or at the locations of instrumented frames or their sidelobes).

5. **Volume versus Bottom Backscatter**

A reasonable hypothesis is that bottom backscatter competes with volume backscatter, biasing the net estimate toward zero. If there were only the direct acoustic path, the bottom backscatter would create a very narrow line at zero Doppler shift, and a line-filter could be used to remove it (as is done with some radar systems). However, here paths including grazing-angle reflections off the surface are also important, introducing small quasi-random Doppler shifts (see figures 6, 7). Thus the net bottom backscatter is a statistical variable with expected Doppler shift of zero and with finite frequency bandwidth. With a little averaging (perhaps as little as 80 ms, or one repeat-sequence-code length; more surely over a wave period), the bottom backscatter intensity should remain statistically steady over time scales of hours. In contrast, volume backscatter (the signal of primary interest) varies on advection time scales (minutes or faster) or surface wave time scales (seconds), as bubbles are injected and advected. This leads to a simple model to describe, evaluate, and correct for the effect. With this model, the ratio of measured to modeled wave variances (cf. Section 4) can be used to deduce an appropriate division of the averaged backscatter intensity into a “bottom component” and a “volume component.”

Exposition of the model requires a brief review of the Doppler processing technique: the Doppler

![Figure 6](image.png)

**Figure 6.** The surface is a shifting blurry mirror with respect to acoustic propagation. Outgoing and return rays can each take direct or reflected paths, resulting in 4 routes out and back. For a source/receiver at “A,” the image location “B” provides a tool for understanding the paths. The locus of points at a fixed time-delay from an impulse transmission has 4 parts: (1) the direct path out and back yields a spherical segment $S_{AA}$ centered on A; (2) the reflected path out and back yields a spherical segment $S_{BB}$ centered on B; (3) the reflected path out and direct return $S_{BA}$ yields an ellipsoidal segment with foci at A and B; and (4) the direct path out and reflected path back $S_{AB}$ yields the same ellipsoidal segment, except that the surface reflection occurs later (on the return trip), by which time the surface may have changed (hence also the net signal phase). Where the segments meet the surface they coincide; at the bottom, the backscattered signal comes from 3 points (actually arcs into the page). If the reflection is weak (e.g., absorbed by the bubble layer), the virtual source strength or receive sensitivity at B is adjusted to match. Additional reflections (e.g., off the bottom) make the picture more complex but straightforward: every pair of real or virtual source/receivers forms another ellipsoid. Finite transmission length and averaging thickens the segments into volumes and bottom arcs into areas.
shift is estimated by an autocovariance technique, using repeat sequence codes (Pinkel and Smith, 1992). The signal is complex-homodyned to a center frequency of zero, and a time-lagged autocovariance formed. The intensity-weighted mean Doppler shift is proportional to the phase on the complex plane of this autocovariance (Miller and Rochwarger, 1972). Figure 8 illustrates the effect of an additive zero-Doppler component, and defines the geometry and terminology used here. As illustrated, there are 5 variables: the desired signal intensity $S$ and its phase $\alpha$, the bottom intensity $B$ (assumed to have zero phase), and the measured intensity $I$ and its phase $\beta$. The real and imaginary parts must separately add up, so there are two equations:

$$I \sin \beta = S \sin \alpha \Rightarrow S = I \left( \frac{\sin \beta}{\sin \alpha} \right)$$

(18)

and

$$I \cos \beta = B + S \cos \alpha \Rightarrow B = I \cos \beta - S \cos \alpha = I \left( \cos \beta - \frac{\sin \beta}{\sin \alpha} \right).$$

(19)

Solution requires one parameter more than the two measured ($I$ and $\beta$). For example, if there were an $a$ priori estimate of the bottom backscatter intensity $B$, an estimate of the signal phase $\alpha$ can be extracted from the measured backscatter parameters $I$ and $\beta$.

Figure 7. Intensity plot from a vertical fan beam in shallow water (data taken near Scripps Pier, 1992). The water was unusually clear, as the wind was under 2 m/s for several days. In addition to a still visible line of high backscatter from the surface, the bottom and its smeared reflection off the surface are visible in this picture, supporting the interpretation illustrated in figure 6. In this picture it appears that the largest integrated return at a fixed range would come from the smeared image of the bottom, reflected off the surface.
Conversely, if there were an \textit{a priori} estimate of the signal phase $\alpha$, 19 shows how to extract an estimate of $B$.

The wave model of section 4 provides estimates of wave orbital velocity variances over the fields of view of the sonars. Rigorous use of those here would be difficult; however, for small angles (velocities resulting in covariance phases less that 1 radian or so), the RMS values can be re-scaled and substituted for the measured maps of angular variance. This can only be applied to data averaged over times sufficient to obtain robust variance estimates (say, 2 minutes or longer). The simple signal model described here provides a way to divide the measured intensity into a component due to bottom backscatter and one due to volume backscatter (both presumably non-negative); see figure 9. The bottom intensities are expected to vary slowly in time, so that rapid variations should appear in the more variable volume component. If so, the model can be inverted, using the measured intensity and a fixed bottom intensity to estimate the volume/bottom backscatter ratio on a ping-by-ping basis. Corrections for the effect can be estimated and applied for every “ping,” improving the response over all time scales. The bottom backscatter contribution to the intensity can be verified and updated periodically using time-averaged wave variance estimates.

6. \textbf{Results and Conclusions}

Data from discrete locations within the PADS viewing area in SandyDuck were used for direct comparisons of velocity estimates. Near-bottom currents were provided by S. Elgar et al., and current profiles in 25-cm vertical bins have been provided by P. Howd for this purpose. Comparisons between PADS and other current measurements is encouraging, with correlations typically in the range of 0.90 to over 0.99 (depending on the measurements being near-surface or near-bottom, and on the existence of stratification).

For lower frequency motions, such as shear-waves and eddies associated with the alongshore shear, the correspondence between near-surface and near-bottom measurements can vary depending on the

\[ \alpha = \arctan 2(I \sin \beta, I \cos \beta - B). \]  \hfill (20)

\textbf{Figure 8.} Illustration of how bottom backscatter contributes to a time-lagged covariance estimate. The addition of a zero-Doppler component $B$ increases the real part of the covariance, but leaves the imaginary part unchanged. In general, this biases the estimate to smaller velocities. For small angles (currents much smaller than the ambiguity velocity) and limited bottom backscatter ($B$ not overwhelming $S$), there is good reason to expect that the effect can be undone. The problem entails 5 variables: three magnitudes $I$, $B$, and $S$, and the two phase angles $\alpha$ and $\beta$. 
stratification. The profile data has been used to evaluate the depth of the strongest correlation with the PADS data, and the correlation with various depth-weighted averages. The depth of measurement most tightly correlated with the PADS estimates is near 1.5 m below the surface (mean with respect to waves, moving with the tide), in line with a priori expectations based on experience in deeper water. Where there is strong vertical mixing, currents near the bottom correlate well with the near-surface currents. In contrast, when there is stratification the correlation can become small, with mean angles over 45° between the top and bottom.

A technique was developed here to make the division between bottom and volume backscatter, and so provide corrected estimates of the Doppler shift due to the volume fraction alone in the acoustic signal. The method depends on independent directional wave information, at minimum a mean

Figure 9. An example of the separation of the total backscattered acoustic intensity field (top panel) into the contributions from the volume signal (lower left panel) and bottom backscatter (lower right panel). Note the high-intensity volume fraction advecting out with a “rip current” near \((x, y) = (1000, 250)\), with little affect on the estimated bottom backscatter relative to the surroundings. Two more patches of higher-backscatter fluid are seen at \((1100, 500)\) and \((900, 500)\); these are advecting to the right, parallel to the shore. Arrows represent fluid velocity estimates; an arrow of length equal to the grid spacing corresponds to 40 cm/s.
direction, period, and directional spread. Wave motions, which penetrate in a predictable way to the bottom in finite-depth water, can be compared more readily than lower frequency motions that may be baroclinic. At surface wave frequencies, cross-spectra show high correlations up to frequencies of about 0.2 Hz. Higher frequency waves have lengths comparable to the 20 m averaging scale of the measurements (i.e. less than 40 m). The wave information could be obtained from the upward sidelobe returns (as suggested by one reviewer), or from an array of pressure sensors on the mounting frame, if the deployment need be self-contained.

For the nearly uniform alongshore beach seen seaward of 3 m depth at Duck, a simple wave propagation model can be used to extend the comparison over the whole area probed by each PADS. From this comparison, it is seen that the velocity transfer function varies over time and space. The response is consistent with interference from bottom backscatter (with near-zero Doppler shift) mixing with a highly variable volume backscatter element (e.g., bubble clouds) that advect with the flow. “Calibration factors” larger than 1 are required to match variances in the surface-wave band from the PADS with those from other instruments, with values generally increasing as the depth decreases (note correlations remain high even so). The transfer function variations are large-scale and slow compared to the waves, so (for example) high-resolution frequency-directional spectra are useful, within a global factor to correct the total variance. In particular, phase information is robust (e.g., location and celerity of wave crests), so that wave propagation and refraction can be rigorously examined.

Error estimates on the uncorrected Doppler velocity estimates at finite signal-to-noise estimates are discussed in a companion paper (Smith, 2001), based on lower bound calculations (Theriault, 1986) and comparisons with real-world performance of “repeat-sequence” coded pulse systems (Pinkel and

**Figure 10.** Velocity vectors with associated error estimates. The fraction of total variance (signal + noise) attributable to error is contoured as shades of gray. For example, if one component of velocity is well estimated and the other is not, this is shaded at the 0.5 level. As for figure 9, a vector of length equal to the grid spacing corresponds to an estimated 40 cm/s fluid velocity.
Smith, 1992). For use in assimilation of the PADS data into a model, the error estimates are simply scaled by the same factor as that applied to the velocity estimates (e.g., see figure 10).

Finally, it’s worth noting that most previous investigations using sound to probe the nearshore took place off the West Coast, in particular at Scripps Beach, San Diego (Dahl, 2001; Farmer et al., 2001; Smith, 1993; Smith and Largier, 1995; Vagle and Farmer, 2001). The steeper bottom slope and lack of sandbars there apparently make bottom interference less of a problem, and moves the acoustic “bubble barrier” due to breaking surf closer to shore. In this sense, the SandyDuck experiment has proven a somewhat more stringent test of the technique than anticipated.

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